EXPLORATION 1 Constructing a 1-Radian Angle

Carefully draw a large circle on a piece of paper, either by tracing around a circular object or by using a compass. Identify the center of the circle (O) and draw a radius horizontally from O toward the right, intersecting the circle at point A. Then cut a piece of thread or string the same size as the radius. Place one end of the string at A and bend it around the circle counterclockwise, marking the point B on the circle where the other end of the string ends up. Draw the radius from O to B.

The measure of angle AOB is 1 rad.

1. What is the circumference of the circle, in terms of its radius r
2. How many radians must there be in a complete circle?
3. If we cut a piece of thread 3 times as big as the radius, would it extend halfway around the circle? Why or why not?
4. How many radians are in a straight angle?

Trigonometry means measurement of triangles

In the space below draw a picture of an angle and label the vertex, initial side and the terminal side.

3:00 position

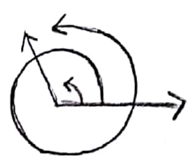
An angle is determined by rotating a ray about its endpoint. The starting position of the ray is the initial side of the angle, and the position after rotation is the terminal side. The vertex is the endpoint of the ray. An angle that fits the coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis is an angle in standard position. Counterclockwise rotation generates positive angles while clockwise rotation generates negative angles.

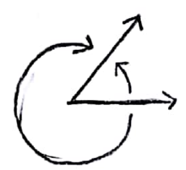
How are angles labeled?

greek letters α β θ

uppercase letters A B C (vertex)

Angles that have the same initial and terminal sides are called coterminal angles. In the space below draw an example of coterminal angles.





A measure of an angle is determined by the amount of rotation from the initial side to the terminal side.

One way to measure angles is in radians. Another way to measure angles is in degrees.

One **radian** is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle.

In other words, $\theta = \frac{s}{r}$ where θ is measured in radians. (Note that $\theta = 1$ when $s = r$.)

$s = r\theta$

Remember, the circumference of a circle is $2\pi r$ units and it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of $s = 2\pi r$. Also recall that there are approximately 6.28 radius lengths in a full circle ($2\pi \approx 6.28$).

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\frac{\pi \; rad}{180°}$.
2. To convert radians to degrees, multiply radians by $\frac{180°}{\pi \; rad}$.

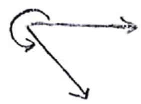
Example 1: Express each of the following angles in radian measure as a multiple of π. (Do not use a calculator.)

a. 420°

$42\cancel{0} \times \frac{\pi}{18\cancel{0}}$

$\frac{42}{18} = \frac{7\pi}{3}$ rad.

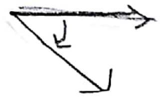
b. 280°



$28\cancel{0} \times \frac{\pi}{18\cancel{0}}$

$\frac{28}{18} = \frac{14}{9}\pi$ rad.

c. $-30°$



$-30 \times \frac{\pi}{180}$

$\frac{-30}{180} = \frac{-\pi}{6}$ rad.

Example 2: Express the following angles in degree measure. (Do not use a calculator.)

a. $\frac{\pi}{9}$

$\frac{\cancel{\pi}}{9} \times \frac{180}{\cancel{\pi}}$

20°

b. $\frac{8\pi}{3}$

$\frac{8\cancel{\pi}}{3} \cdot \frac{180}{\cancel{\pi}}$

480°

c. 3 radians

$3 \cdot \frac{180}{\pi} = \frac{540}{\pi}$

171.89°

to

Arc Length

For a circle of radius r, a central angle θ intercepts an arc length s given by $s = r\theta$ where θ is measured in radians.

Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

$S = r\theta$

$\theta = \frac{S}{r}$

$r = \frac{S}{\theta}$

Example 3:

r S convert

A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 140°.

$140 \times \frac{\pi}{180} = \frac{14}{18} = \frac{7\pi}{9}$ rad. $= \theta$

$S = r\theta$

$10\left(\frac{7\pi}{9}\right) = \frac{70\pi}{9}$ in.

Linear and Angular Speeds

Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** v of the particle is:

$$\text{Linear speed } v = \frac{\textit{arc length}}{\textit{time}} = \frac{s}{t}.$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** ω (the lowercase Greek letter omega) of the particle is:

$$\text{Angular speed } \omega = \frac{\textit{central angle}}{\textit{time}} = \frac{\theta}{t}.$$

Example 4:

circumference cm/sec

The second hand of a watch is 1.3 centimeters long. Find the linear speed of the tip of this second hand as it passes around the watch face.

$C = 2\pi r$

$2(\pi)(1.3) = \frac{2.6\pi \text{ cm}}{\cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ sec}} \rightarrow 2.6\pi \div 60 = 0.136 \text{ cm/sec}$

Example 5:

The circular blade on a saw rotates at 4200 revolutions per minute. (rpm) each rev. $= 2\pi$ rad

a. Find the angular speed in radians per second.

439.823

$\frac{4200 \cancel{\text{rev}}}{\cancel{\text{min}}} \times \frac{2\pi \text{ rad}}{1 \cancel{\text{rev.}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ sec}} = \frac{8400\pi \text{ rad}}{60 \text{ sec}} = 140\pi \text{ rad/sec} \approx$

b. The blade has a radius of 6 inches. Find the linear speed of a blade tip in inches per second.

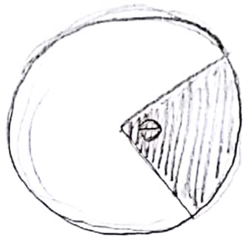
1 radian = 6 in along the arc.

$\frac{140\pi \cancel{\text{rad}}}{\text{sec}} \times \frac{6 \text{ in}}{1 \cancel{\text{rad}}}$ 840π in/sec ≈ 2638.938 in/sec

A <u>sector</u> of a circle is a region bounded by two radii of the circle and their intercepted arc.

Area of a Sector of a Circle
For a circle of radius r, the area A of a sector of the circle with central angle θ is: $$A = \frac{1}{2}r^2\theta$$ where θ is measured in radians.

π



Example 6:

radius

A sprinkler on a golf course is set to spray water over a distance of 75 feet and rotates through an angle of 135°. Find the area of the fairway watered by the sprinkler.

$$135 \times \frac{\pi}{180} = \frac{3\pi}{4} \text{ rad.} = \theta$$

$$A = \frac{1}{2}(75)^2(3\pi/4) = \boxed{6626.797 \text{ ft}^2}$$